# Pipe flow hydraulics - Handout



## Lecture 2. Pipe flow hydraulics

### 1 How do we define hydraulic Gradient and Energy Gradient?

- We need two further concepts, the *Hydraulic Gradient* and the *Energy Gradient*, which we can apply to pipe and channel systems. Here we will always assume that the flow is turbulent, so that the Coriolis averaging coefficient can be assumed to be equal to 1. See the figure below.
- Hydraulic gradient is the plot of potential energy, i.e. $\frac{p}{\rho g} + z = h_p$ against distance along the pipe (horizontal distance usually, which for realistic small pipe gradients will be much the same as the slope distance). This quantity is usually indicated with *h.*
- Energy gradient is the hydraulic gradient plus the velocity head, i.e. $H = \frac{u^2}{2g} + \frac{p}{\rho g} + z$ against distance. This is the total head and is usually indicated with *H.*
- We sometimes call the energy gradient the *friction slope*. You may see it notated $S_F$ or *j*;
- The motion of a fluid implies that a distributed dissipation *j* per unit length *x,*
  $j = -\frac{\partial H}{\partial x}$
  occurs, causing for steady motion in pipes of constant diameter a constant decrease of both the piezometric and total head, i.e. of the total energy of the fluid (Figure 1). The total energy loss $\Delta H_d$ over a length *l* due to distributed dissipation is therefore
  $\Delta H_d = i\ l.$
- Clearly for steady flow in a pipe of constant cross section, the two gradient lines will be separated by a constant distance given by $\frac{\bar{u}^2}{2g}$. However if the pipe section changes, $\bar{u}$ will change so the separation of the two lines will also change.
- If a stand pipe were inserted into the pipe system (at right angles to the flow direction) the water would rise up to the level of the hydraulic gradient.
- For flow in pressurized pipes (such as most water mains and storm water sewers in storm conditions) the hydraulic gradient is not the same as the geometric gradient. It is the head difference along the pipe divided by the pipe length, *not* the elevation difference divided by the length.

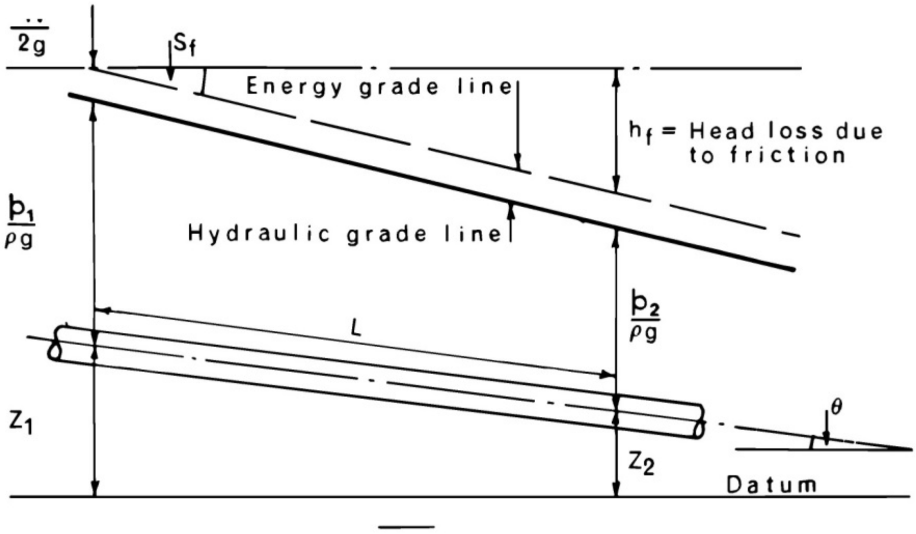



Figure 1

## 2 Distributed vs localized friction losses

- In the case of fully developed turbulent motion within rough pipes (i.e. $f=f(k/d)$) the Coriolis coefficient $\alpha_c$ can be assumed equal to 1 and several practice formulas are useful to approximate the distributed friction losses *i*
  DARCY-WEISBACH

  $j = \beta \frac{Q^2}{d^m}$ *where if* { $\beta = \beta'$ => $m = 5.33$ ; $\beta = \beta_i(d)$ => $m = 5$

  CHEZY

  $\bar{u} = C\sqrt{R\,j}$ *where*

  $C = \frac{87}{1+\frac{\gamma_B}{\sqrt{R}}}$ *Bazin*

  $C = \frac{100}{1+\frac{m_K}{\sqrt{R}}}$ *Kutter*

  $C = c_{GS} R^{1/6}$ *Gaucler-Strikler*

  These practical formulas are useful because they are explicit and therefore of easier use compared to implicit ones like the Collebrook & White one, etc.

- Local losses due to bends, restrictions, expansions, etc appear in both lines as vertical drops. As such localized energies losses $\Delta H_l$ can be expressed as a fraction $\xi$ of the kinetic energy of the fluid in the reach downstream the variation in the pipe geometry $\Delta H_l = \xi \frac{U_1^2}{2g}$;
  Similarly, energy inputs such as pumping appear as vertical rises.

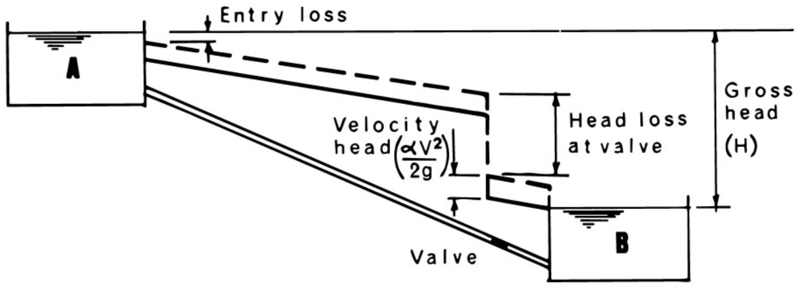



Figure 3

# 3 Various (short) pipeline configurations

- This is a reprise of the second year pipe flow example for revision purposes. See notes in class. As an example, let us consider the graphic representation of the energetic content of the stream in steady pipe flow;
- They key point is balancing head available to drive the flow, either from elevation up a hillside or from a pump, with the friction losses which increase as the flow goes faster.

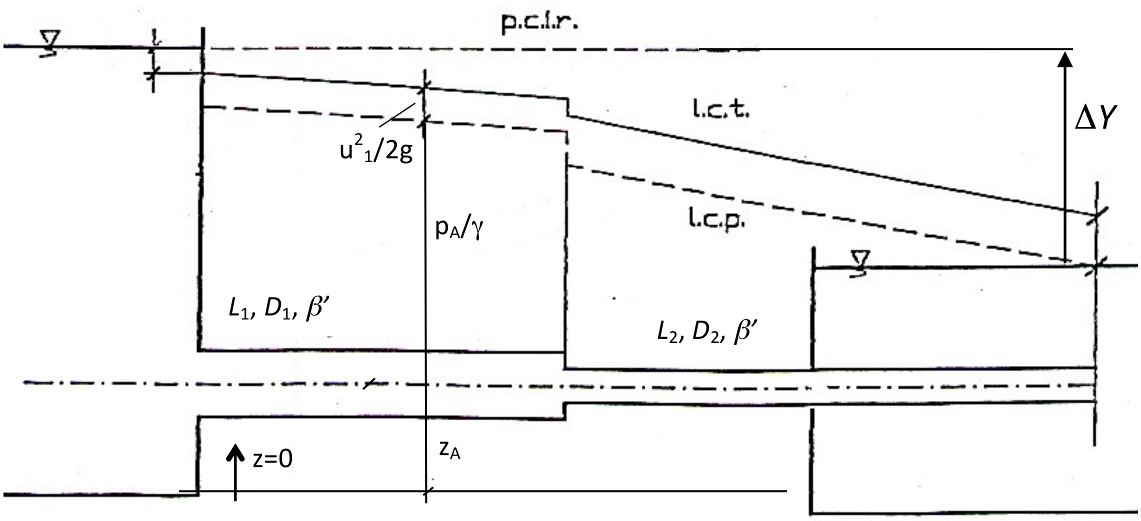
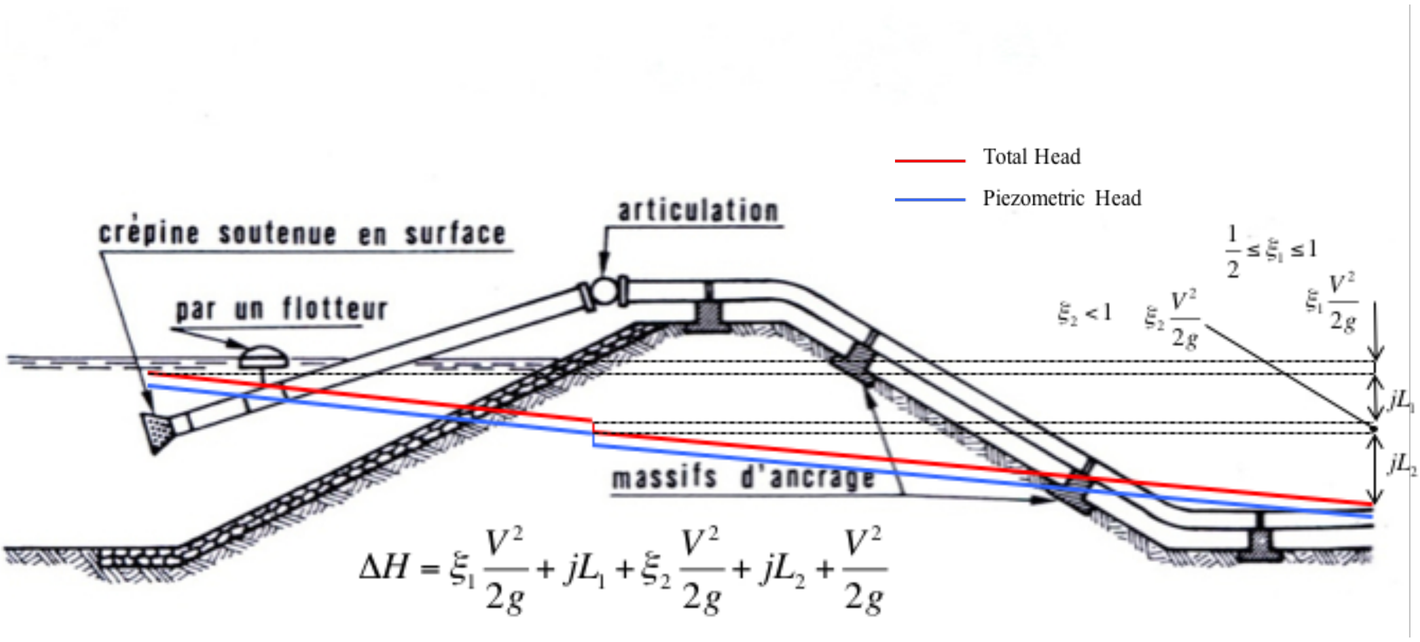



Figure 4

Energy balance eqaution $\Delta Y = 0.5\frac{u_1^2}{2g} + \beta'\frac{Q^2}{d_1^{5.33}}L_1 + \xi\frac{u_1^2}{2g} + \beta'\frac{Q^2}{d_2^{5.33}}L_2 + \frac{u_2^2}{2g}$

SIPHON FLOW

This is another interesting scheme very often used in irrigation plants




## 4 Verification vs design problems

- Notice, that by using the continuity equation for the pipe, $Q = u\frac{\pi D^2}{4} = const$, the energy balance equation can be rewritten in terms of flow rate, $Q$ thus allowing for calculating the flow rate once the geometry of the pipe system (lengths, diameters, restrictions, etc), the pipe roughness and the available energy are known (verification problem). Similarly, if the flow rate is known, one can compute the diameter to convey it with the assigned available energy (design problem). We can summarize the two problems as

| GENERAL VERIFICATION PROBLEMS | GENERAL DESIGN PROBLEM |
|---|---|
| $\Downarrow$ | $\Downarrow$ |
| KNOWN: *D, L, i,* $\varepsilon$ or equivalent roughness | KNOWN: *Q, L, i,* $\varepsilon$ or equivalent roughness |
| $\Downarrow$ | $\Downarrow$ |
| UNKNOWN: effective flow rate *Q* passing within the pipe of diameter *D* with the established dissipation per unit lengh *i* | UNKNOWN: best diameter *D* (pipe size) to convoy the flow rate *Q* with the established dissipation per unit length *i* |

## 5 Trunk Main Design

- Having reminded ourselves of the fundamentals of fluid flow in pipes we can design a gravity flow main such as might be used to supply water over a downhill route.
- A typical arrangement is shown in the figure below.
- Note that the pipe follows the contours. This saves excessive trenching.
- Provided that the outlet is lower than the inlet and the pipe flows full, the short uphill sections do not matter.
- However in order to fill the pipe at start up, there must be air valves at the top of the humps.
- Similarly washout valves are provided at local low points to allow drainage for maintenance.
- The figure shows local energy losses related to bends, valves etc.
- There will also be structural requirements, not covered in this course.
- For example, a bend will result in a force as can readily be determined by applying the momentum equation. In such cases *thrust blocks* may be provided to ensure the pipe remains stable.
- Break-pressure tanks may be provided (not shown) if the head loss over the pipe is far greater than needed for the flow. These are small tanks with a free surface. The inlet to them is controlled by a float valve, so they do not overflow, and the outlet is gravity fed. Such tanks 'reset' the pressure in the pipe to zero.

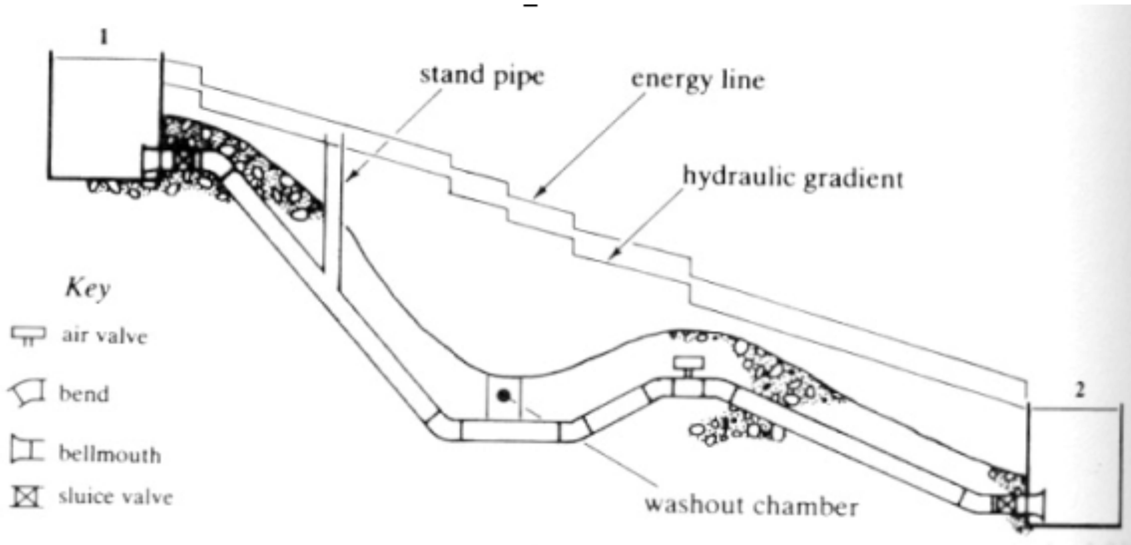



Figure 5

## 6 Hydraulically-long pipes (pipelines)

Let us study now a pipe hydraulically long (pipeline) and consider only the piezometric A-A'. Moreover the line B-B' represents the piezometric referred to the absolute pressure. If the fluid being considered is water then the distance between the two lines is equal to $p_{atm}/\gamma \approx 10.33$ m (Figure 6).

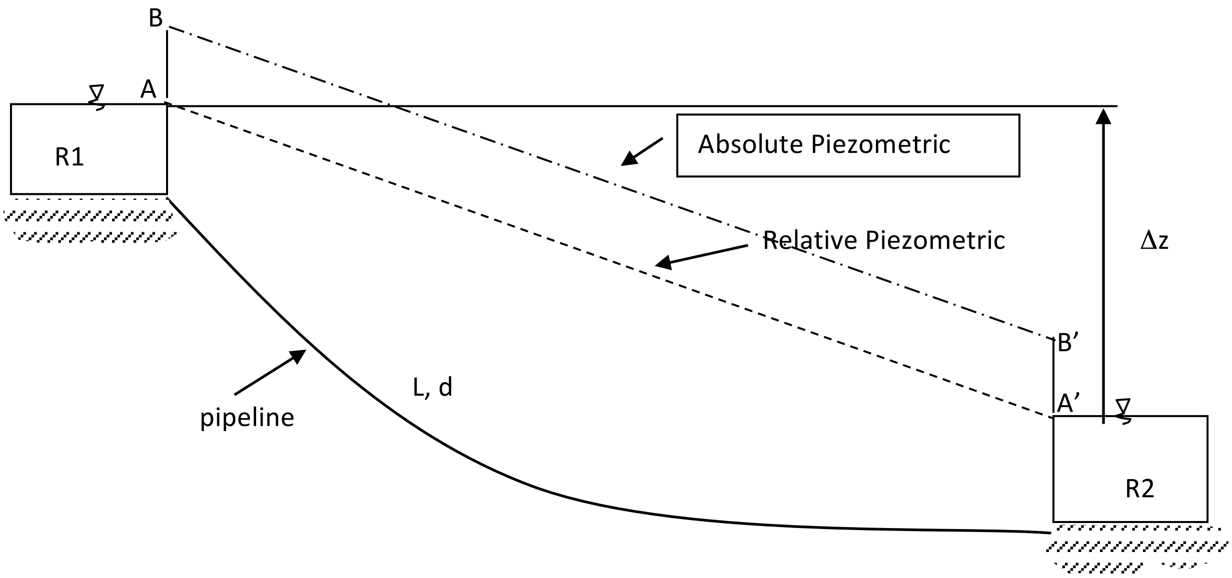



Figure 6. Sketch of a pipeline connecting two reservoirs. For steady flow the energy dissipation is represented by the slope of the piezometric line.

We are interested in quantifying the ratio between the length of the pipe, L and the diameter, d, such that the kinetic term can be neglected in the calculation compared to distributed friction losses. In other words, energies losses due to localized dissipation in presence of geometrical discontinuity as long as the kinetic term (which is usually relatively small, i.e. few m/s) can be neglected (if compared to distributed dissipation) if the pipe is 'sufficiently long'. In this case both the lines of the total and piezometric heads will coincide, and it is useful to work only considering the piezometric head of the stream (Figure 6).

For pipeline systems, the kinetic and energy losses that have order of magnitude $\frac{\bar{u}^2}{2g}$ can be neglected by accepting an approximate solution, i.e. involving a small error. The equation of motion for the flow within the pipelines of Figure 6 is therefore

$$\Delta z = j\,L. \tag{9}$$

We show now which is the condition that allows to define a pipe as *hydraulically long.*

Let us compare a localized dissipation of magnitude $\frac{\bar{u}^2}{2g}$ to an equivalent distributed one over a generic length $L^*$, i.e.

$$\frac{\bar{u}^2}{2g} = iL^*.$$

Supposing to express the distributed energies losses by means of the Chezy formula, we have

$$i = \frac{\bar{u}^2}{C^2 R},$$

and considering that for circular pipes R=*d*/4, and using the definition of the friction factor

$f = \frac{dj}{\frac{\bar{u}^2}{2g}}$, we obtains the following relations for *C*

$$C^2 = \frac{8gL^*}{d} \qquad \text{and} \qquad C^2 = \frac{8g}{f},$$

which lead to the result

$$L^* = \frac{D}{f}. \tag{13}$$

Considering a common value for the friction factor, *f*=0.02, we have that along a reach of length $L^* \approx 50\ d$ the stream dissipates an energy equal to the kinetic term $\frac{\bar{u}^2}{2g}$.

Definition: A pipe is called 'hydraulically long', regardless the presence of a weak discontinuity in the geometry which induces a localized head loss, if the total pipe length is at least $L > 20\ L^*$ , i.e. L > 1000*d.*

**Example**

Let us now calculate which should be the effective length of a pipe in order to consider it as hydraulically long.

We suppose to accept an approximation of the exact solution of about 4%, i.e 0.04.

The equivalent length *L* thanks to which we would neglect the velocity head, in order to solve the problem with the chosen approximation, can be expressed as a number *n* of time the pipe diameter *d*, i.e. *L=nd* .

Therefore we have that

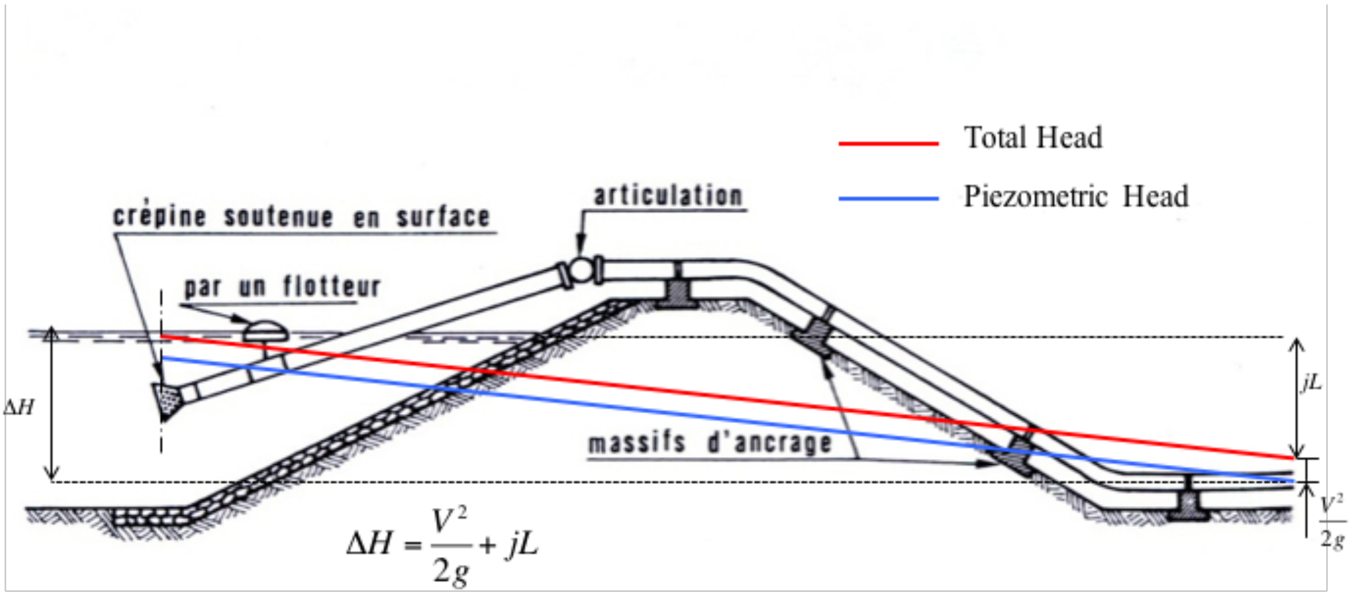
$$\frac{\bar{u}^2}{2g} \leq 0.04Lf\,\frac{\bar{u}^2}{2gd} = 0.04ndf\,\frac{\bar{u}^2}{2gd},$$

and, as a consequence

$$n = \frac{1}{0.04f}.$$

Assuming now a friction factor f=0.025, the result is therefore *n*=1000. This means that for this system if one neglects the velocity head in the calculation, the result will have an approximation of the 4% only if the pipe length is about 1000 times the pipe diameter. Of course the length of the pipe should be longer as more as higher is the number and kind of the localized head losses.

OPEN SIPHON AS A PIPELINE




## 7 Altimetry problems for pipelines

We study now the simple situation of two reservoirs connected by a pipeline which develops on soils having different morphological levels (Figure 7). This is instructive in order to understand how the flow will take place, and whether it may occur spontaneously or has to be triggered.

Since we deal with a pipe hydraulically long (pipeline) we neglect the velocity head and consider only the piezometric A-A'. The line B-B' represent the piezometric referred to the absolute pressure.

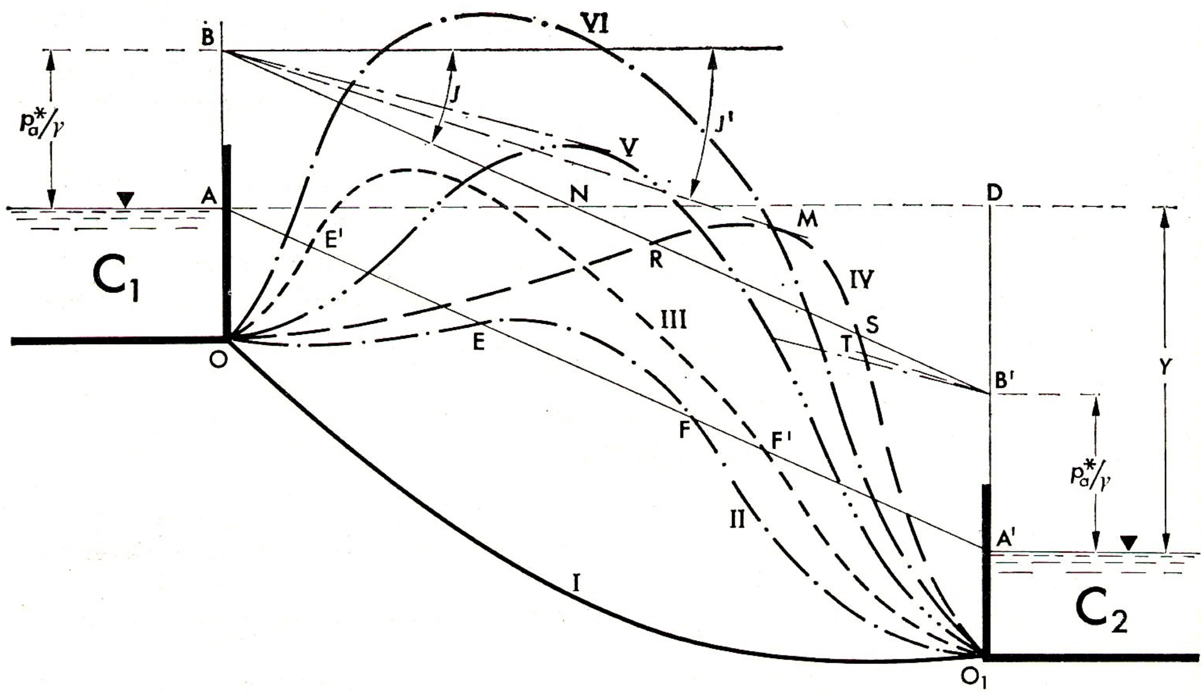



Figure 7. Sketch of a pipeline built over soils with different morphology.

NOTE: For all the cases both the piezometric relative and absolute do not change

Case I. The motion trigs off spontaneously and the pipe is subjected to a positive pressure.

Case II. The pipeline 'crosses' the piezometric relative, so the reach E-F has a pressure negative. The motion trigs off spontaneously.

Case III. The pipeline crosses the piezometric relative , so the reach E'-F' has a pressure negative. The motion doesn't trig off spontaneously since the pipeline has also a reach above the reservoir C1 level. The motion must be trigged off artificially, for example by suction or by filling the pipe.

Case IV. The pipeline crosses both the piezometric relative and absolute, but never goes above the reservoir C1 level. In normal condition the reach R-S would have a pressure negative less then the atmospheric pressure. This is not possible so the system spontaneously adequate itself by changing the slope *J* of the piezometric lines in the new configuration B-M (*J'*). The total stream flow will be less that one would have with a slope A-A' oer BB'. However since the pipe is always below the Reservoi C1 level the motion trigs off spontaneously.

CASE V. Same as CASE IV but the motion has to be trigged off.

CASE VI. No configurations of motion are possible since the pipeline has a reach higher than the level of C1 referred to the absolute piezometric.

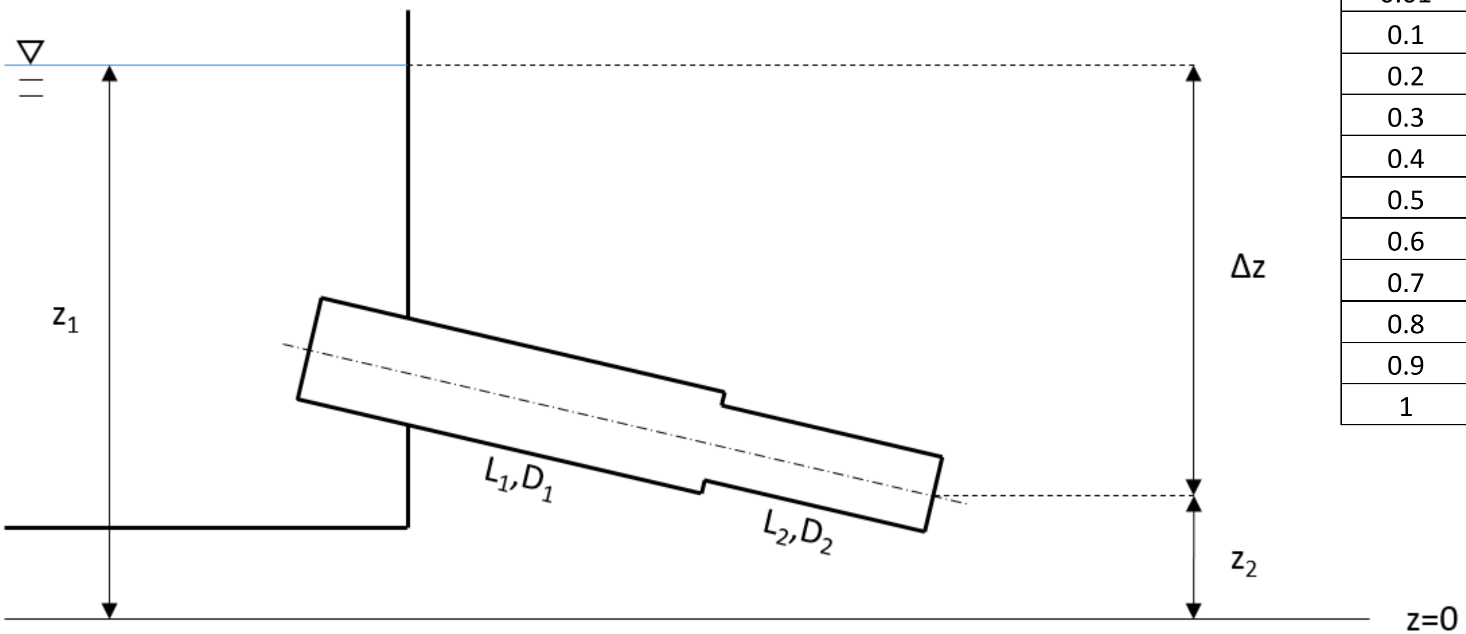
# Example 1: hydraulic verification problem

We consider a conduct constituted by two pipes of diameter $D_1$ and $D_2$, of length $L_1$ and $L_2$, and with diameter dependent Darcy coefficients $\beta_1$ and $\beta_2$.

Knowing the altitude of the water level $z_1$ in the tank and the altitude of the center of the section at the end of the pipe $z_2$, determine the discharge Q at the outlet of the pipe.

Draw the Hydraulic grade line and the Energy grade line.

| $D_2/D_1$ | ξ |
|---|---|
| 0.01 | 0.5 |
| 0.1 | 0.477 |
| 0.2 | 0.452 |
| 0.3 | 0.425 |
| 0.4 | 0.396 |
| 0.5 | 0.358 |
| 0.6 | 0.31 |
| 0.7 | 0.243 |
| 0.8 | 0.166 |
| 0.9 | 0.086 |
| 1 | 0 |




$L_1$ = 45 m ; $L_2$ = 25 m ; $D_1$ = 470 mm ; $D_2$ = 300 mm ; $z_1$ = 10 m ; $z_2$ = 5 m

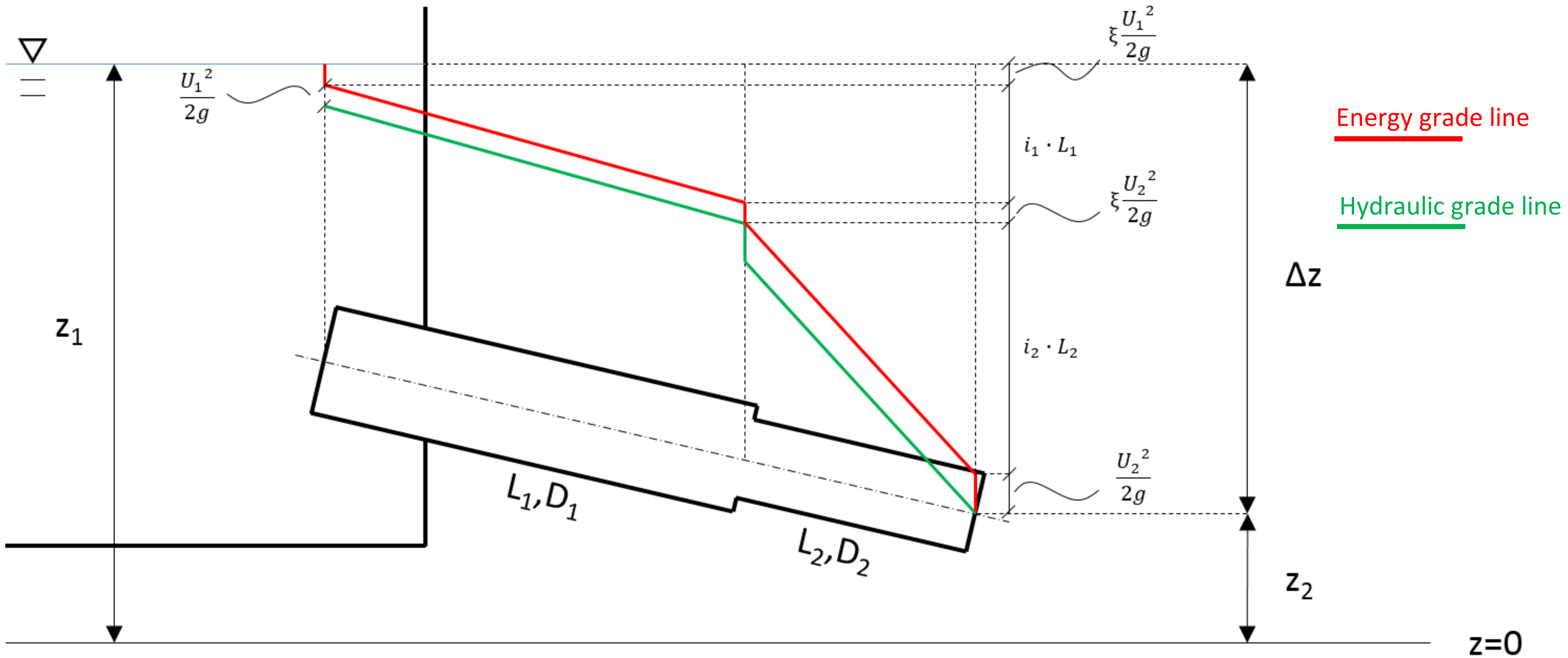
## Solution

Steps:

1) Express the terms of the energy balance as a function of the discharge Q:

   i. the minor loss at the inlet of the pipe
   ii. the minor loss at the connection of the two pipes
   iii. the linear head loss due to friction in each pipe
   iv. the velocity head loss at the outlet of the second pipe

2) Compute the discharge Q knowing that the total head loss is equal to Δz.




$$\Delta z = \frac{U_1^2}{2g} + \xi\frac{U_2^2}{2g} + i_1 \cdot L_1 + i_2 \cdot L_2 + \frac{U_2^2}{2g}$$ Energy balance

| $D_2/D_1$ | ξ |
|---|---|
| 0.01 | 0.5 |
| 0.1 | 0.477 |
| 0.2 | 0.452 |
| 0.3 | 0.425 |
| 0.4 | 0.396 |
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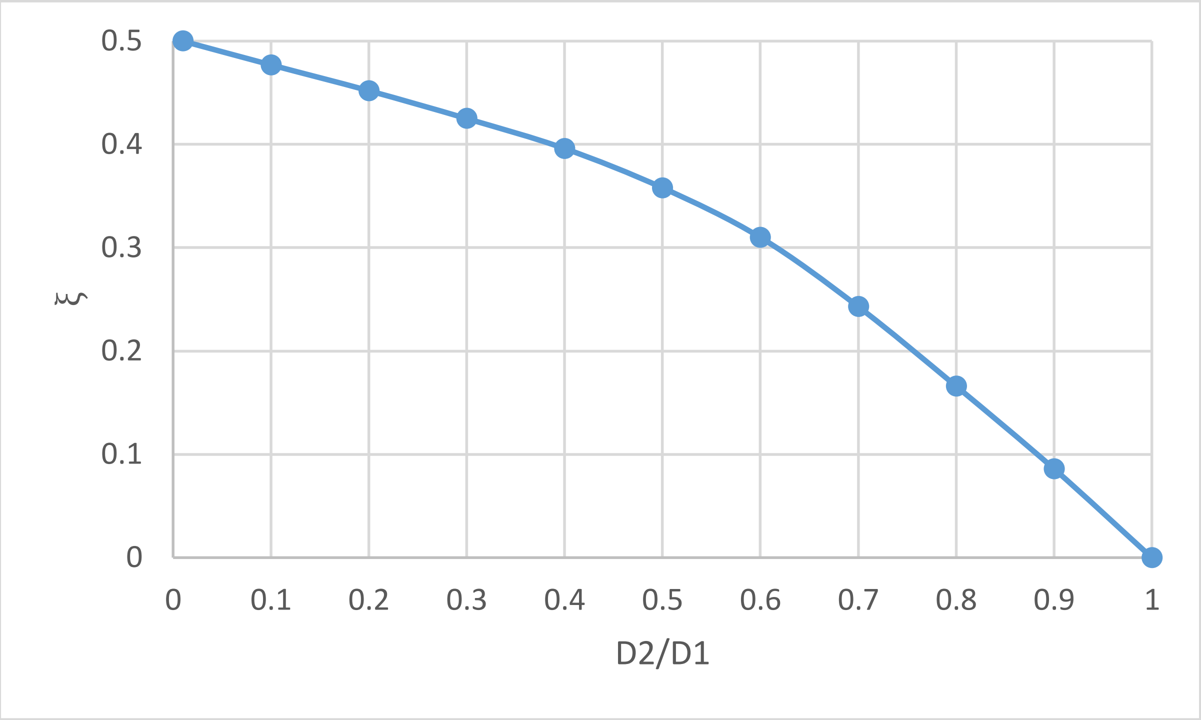
| 0.8 | 0.166 |
| 0.9 | 0.086 |
| 1 | 0 |




- When the roughness coefficient β is unknown, it can be computed as a function of the pipe diameter D with the following empirical function: $\beta(D) = 2 \cdot (0.00164 + \frac{0.000042}{D})$.

In this case, the following Darcy-Weisbach equation can be used to compute the distributed loss: $i = \beta(D) \cdot \frac{Q^2}{D^5}$.

1) Express the terms of the energy balance as a function of the discharge Q:

i. the minor loss at the inlet of the pipe

➔ $\xi \frac{{U_1}^2}{2g} = 1 \cdot \frac{Q^2}{2g \cdot \left(\frac{\pi \cdot {D_1}^2}{4}\right)} = 1.695\, Q^2$

ii. the minor loss at the connection of the two pipes

$\frac{D_2}{D_1} = 0.6383$, using the chart we find $\xi = 0.284$.

➔ $\xi \frac{{U_2}^2}{2g} = 0.284 \cdot \frac{Q^2}{2g \cdot \left(\frac{\pi \cdot {D_2}^2}{4}\right)} = 2.9\, Q^2$

iii. the linear head loss due to friction in each pipe

The roughness coefficient is calculated for each pipe as follows:

$$\beta_1 = 2 \cdot \left(0.00164 + \frac{0.000042}{0.47}\right) = 0.0035$$

$$\beta_2 = 2 \cdot \left(0.00164 + \frac{0.000042}{0.3}\right) = 0.0036$$

Then, the Darcy-Weisbach equation is used to calculate the head loss:

➔ $i_1 \cdot L_1 = \beta_1 \cdot \frac{Q^2}{{D_1}^5} \cdot L_1 = 6.87\ Q^2$

➔ $i_2 \cdot L_2 = \beta_2 \cdot \frac{Q^2}{{D_2}^5} \cdot L_2 = 37.04\ Q^2$

iv. the velocity head loss at the outlet of the second pipe

➔ $\frac{{U_2}^2}{2g} = \frac{Q^2}{2g \cdot \left(\frac{\pi \cdot {D_2}^2}{4}\right)^2} = 10.2\ Q^2$

2) Compute the discharge Q knowing that the total head loss is equal to Δz.

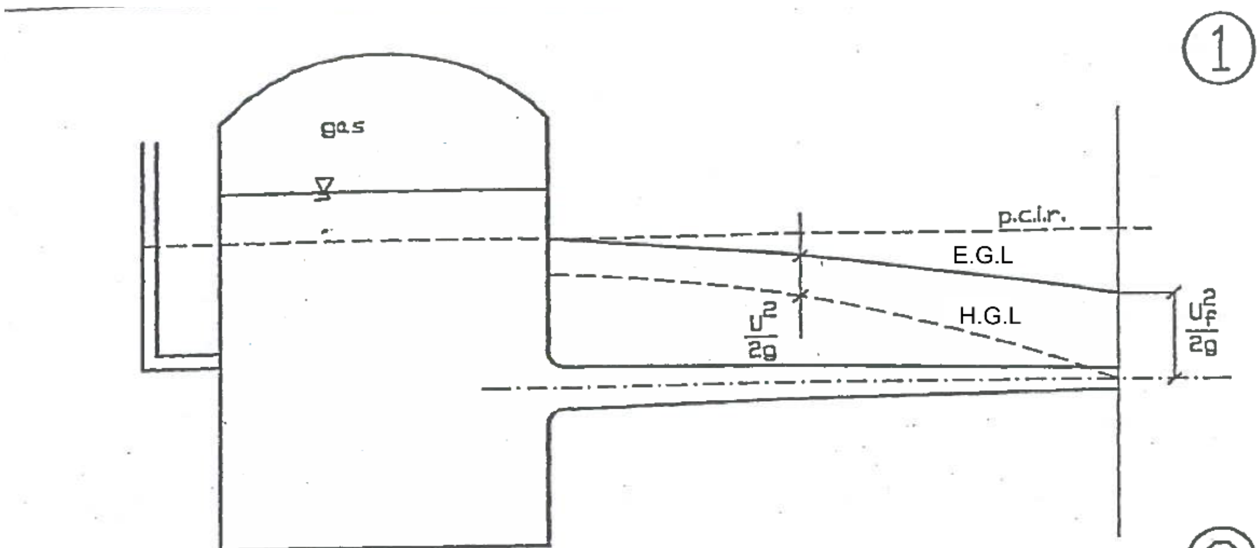
$$\Delta z = \frac{{U_1}^2}{2g} + \xi \frac{{U_2}^2}{2g} + i_1 \cdot L_1 + i_2 \cdot L_2 + \frac{{U_2}^2}{2g}$$

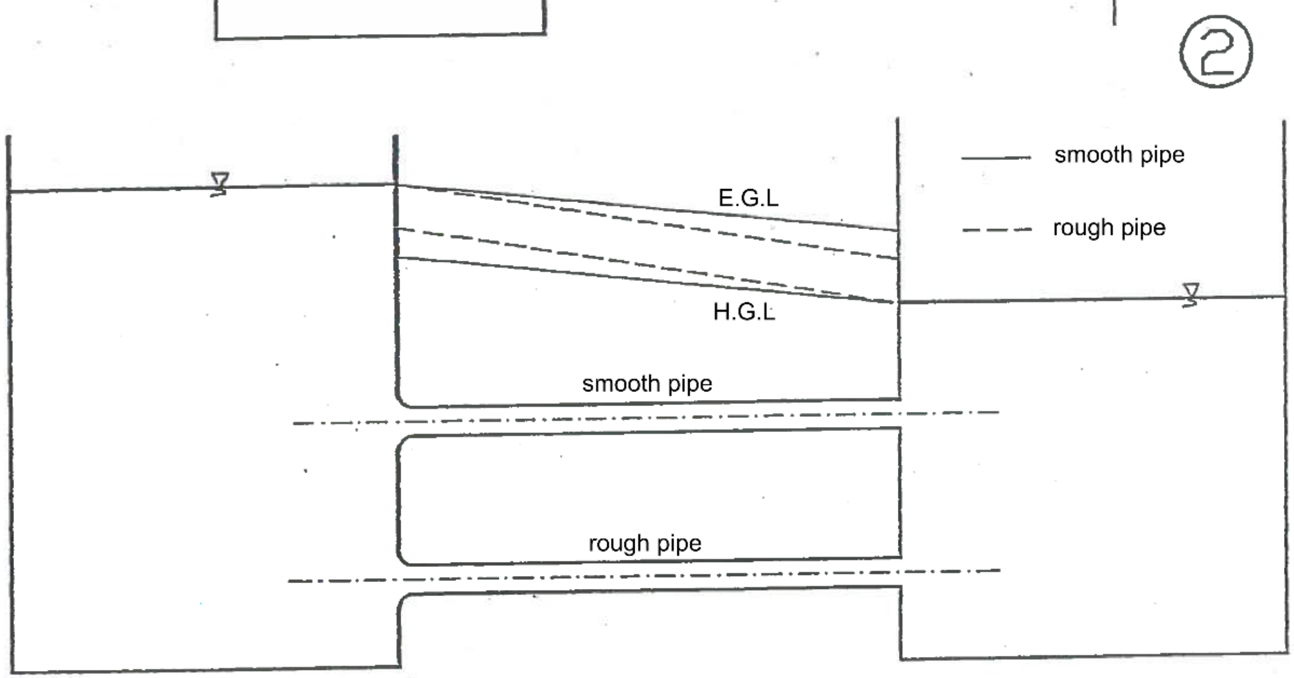
$$\Delta z = z_1 - z_2 = Q^2 \cdot (1.695 + 2.9 + 6.87 + 37.04 + 10.2)$$

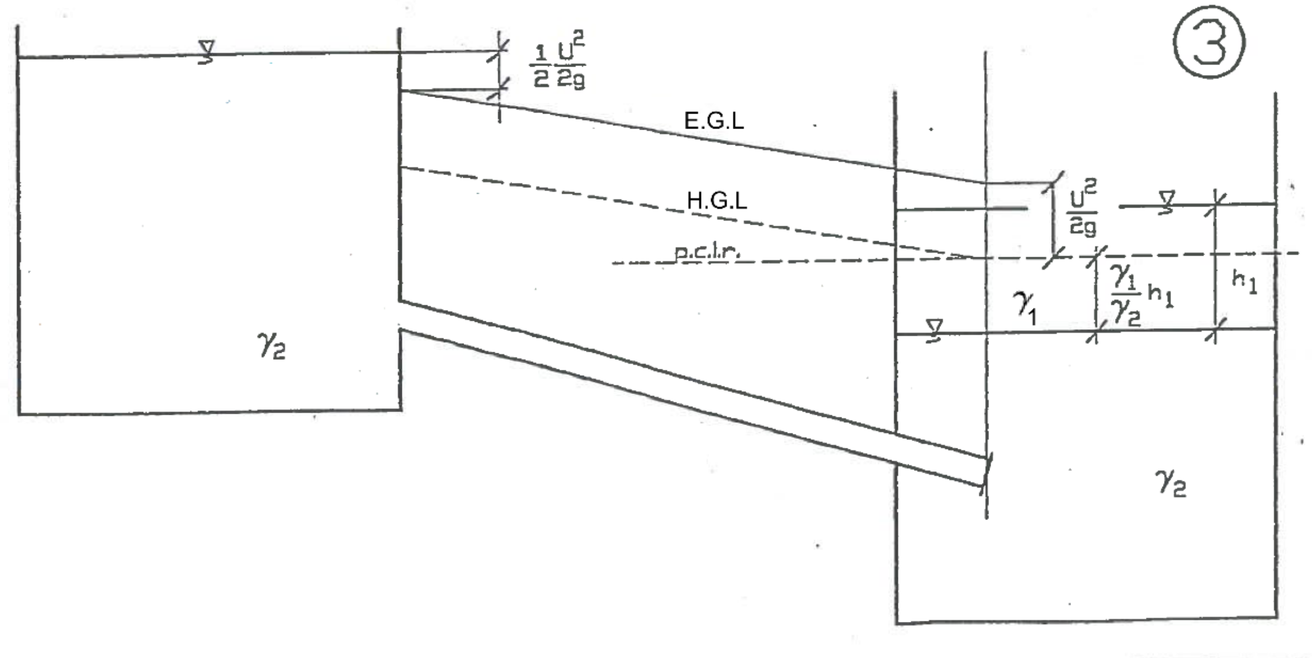
$$Q = \sqrt{\frac{10 - 5}{(1.695 + 2.9 + 6.87 + 37.04 + 10.2)}}.$$

➔ *Q = 0.29 m3/s*

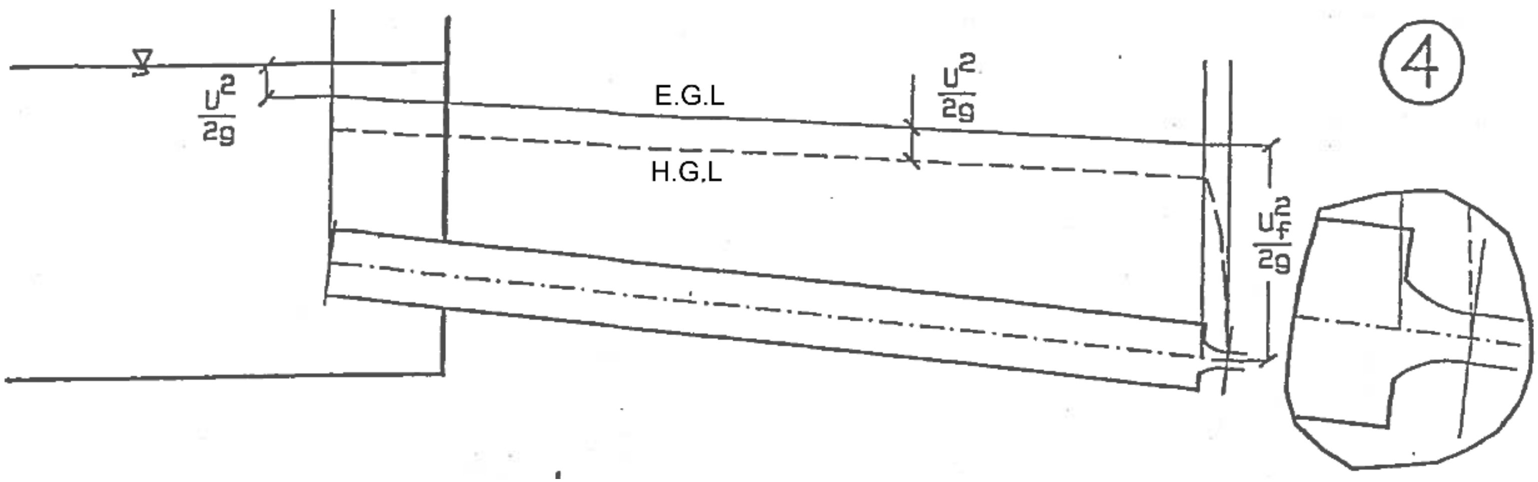
# Drawing EGL and HGL for some notable cases





③
1/2 U²/2g
E.G.L
H.G.L
p.c.l.r.
U²/2g
γ1/γ2 h1
h1
γ1
γ2
γ2

④
U²/2g
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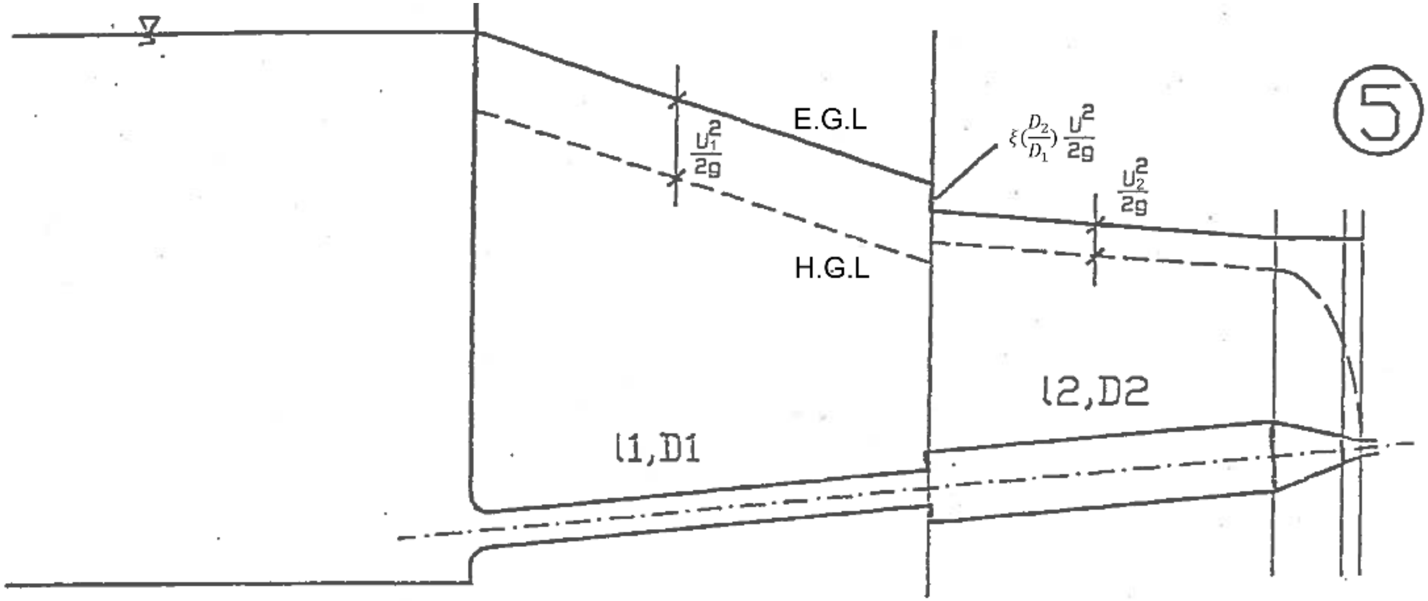
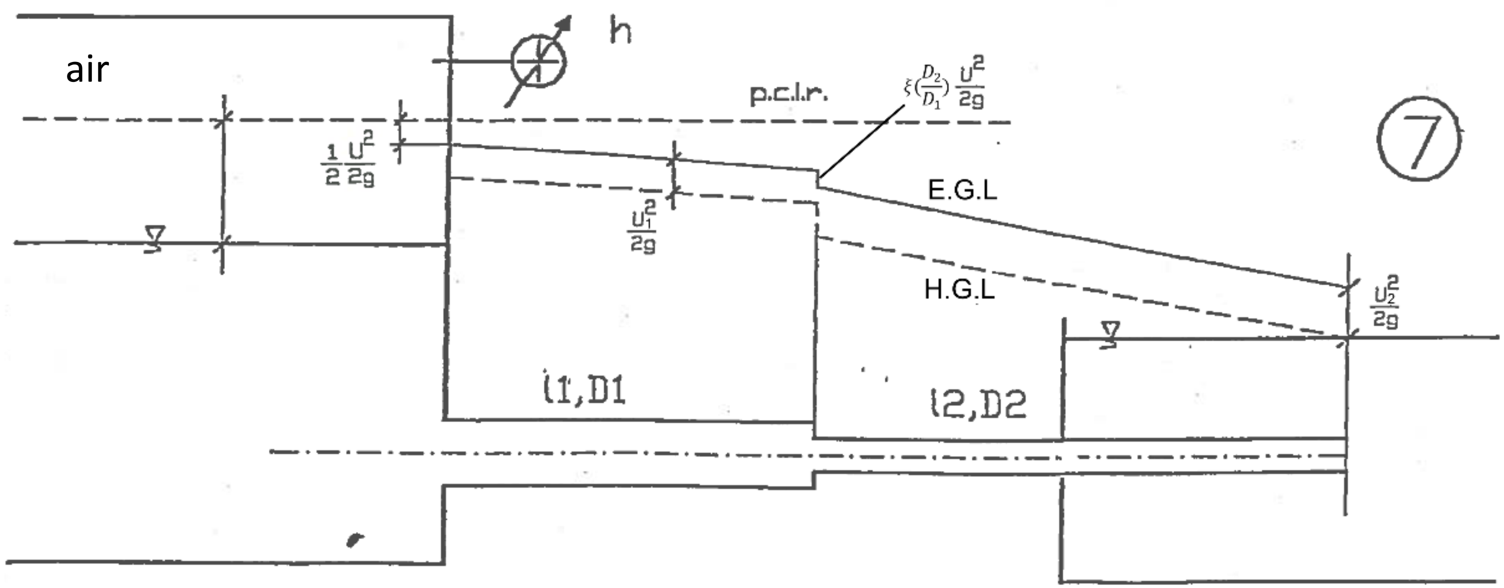

E.G.L
H.G.L
$\frac{U_1^2}{2g}$
$\xi(\frac{D_2}{D_1})\frac{U^2}{2g}$
$\frac{U_2^2}{2g}$
l1,D1
l2,D2
5


air
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l1,D1
l2,D2
7